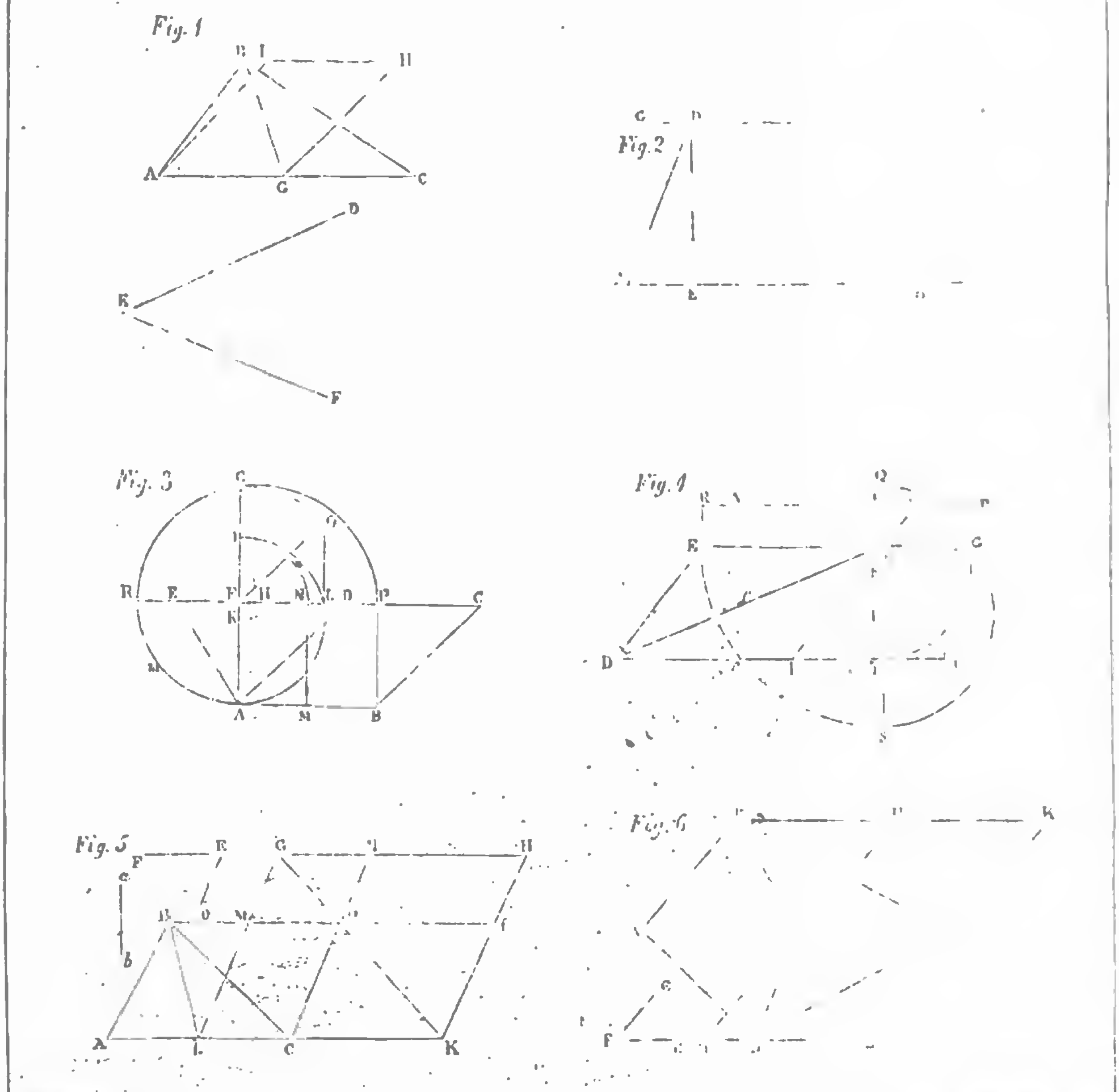
CONVERSION OF PLANE FIGURES.



PRACTICAL GEOMETRY.

THE CONVERSION OF PLANE FIGURES.*

PROBLEM.—*To construct a parallelogram that shall contain an area equal to a given right-lined triangle, and have one of its angles equal to a given angle; that is, to convert a given triangle into a parallelogram of equal area, having one of its angles given.*

Example.—Let ABC (Fig. 1) be the given triangle, and DEF the given angle; it is required to constitute a parallelogram that shall contain the same area as the right-lined triangle ABC, and have an angle equal to the given angle DEF.

Construction.—Divide AC, the base of the given triangle ABC, into two equal parts in the point G, and at the point A, one extremity of the base AC, make the angle GAI equal to the given angle DEF; then, through G, the middle of the base AC, draw GH parallel to AI, and through B, the vertex of the triangle ABC, draw the straight line BIH parallel to the base AC, and meeting the sides AI and GH in I and H, and AIHG will be the parallelogram required, containing an area equal to the given triangle ABC, and having an angle equal to DEF.

Demonstration.—Draw the straight line BG connecting the vertex of the given triangle at B, with the middle of the base at G; then, because AG is equal to GC by construction, the triangle ABG is equal to the triangle GBC, for they stand upon the equal bases AG and GC, and are between the same parallels AC and BH; therefore, the whole triangle ABC is double of the triangle ABG; but the parallelogram AIHG is double of the triangle ABG, for it stands upon the same base, AG, and is between the same parallels AG and HI; consequently, the parallelogram AIHG is equal to the triangle ABC, for things that are double of the same thing are equal to one another, and it has, moreover, the angle GAI equal to the given angle DEF.

Note.—When the given angle DEF is a right angle, the construction is somewhat simplified, as in that case we have only to erect perpendiculars at the points A and G to meet BI drawn through B parallel to the base AC, and the thing is done.

PROBLEM.—*To construct a right-angled parallelogram that shall contain an area equal to a given oblique-angled one; or, in other words, to convert an oblique-angled parallelogram, or rhomboid, into a right-angled parallelogram of equivalent area.*

Example.—Let ABCD (Fig 2.) be a rhomboid or oblique-angled parallelogram; it is required to convert the oblique-angled parallelogram ABCD into a right-angled one containing the same area.

Construction.—From D and C, the angles adjacent to one side of the given figure, let fall the perpendiculars DE and CF upon AB the opposite side, the one of them, DE meeting AB within the figure as at E, and the other, CF, meeting AB produced at F; then is DEFC the rectangular parallelogram required, being equal in area to the oblique-angled parallelogram ABCD given.

Demonstration.—Produce CD to G, then, since AD is parallel to BC, and GC meets these parallels in the points at D and C, the exterior angle ADG is equal to the interior and opposite angle BCG; and because DE and CF are each of them perpendicular to AB and its production, they are also perpendicular to GC, which is parallel to AB; consequently, the angles EDG and FCG are right angles; but the part ADG has been shewn to be equal to the part BCG, and, therefore, the remainder ADE is equal to the remainder BCF, and the triangles ADE and BCF are equal and similar, so that the part ADE, which is cut off from the rhomboid or oblique-angled parallelogram ABCD by the perpendicular DE, is equal to the part BCF, which is similarly applied to it by the perpendicular CF, and the whole EFCD

* See also "Geometry of the Square," p. 484.